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Section 2.3 Fractional Exponents:

Please see corrections

1. Simplify each of the following expressions:

$a)\ 16^{\frac{1}{2}}$ 4

$b)\ 100^{\frac{3}{2}}$ 1000

$c)\ 64^{\frac{1}{3}}$ 4

$d)\ 27^{\frac{2}{3}}$ 9

$e)\ (-27)^{\frac{1}{3}}$ -3

$f)\ 49^{\frac{3}{2}}$ 343

$g)\ (-1000)^{\frac{2}{3}}$ 100

$h)\ \left(16^{\frac{1}{2}}\right)^{\frac{3}{2}}$ 8

$i)\ \left(1000^{\frac{2}{3}}\right)^{\frac{1}{2}}$ 10

$j)\ \left(9^{\frac{1}{2}}\right)^{\frac{5}{2}}$ $3^{\frac{5}{2}} = 9\sqrt{3}$

$k)\ \left(81^{\frac{3}{4}}\right)^{\frac{1}{2}}$ $=(3^3)^{\frac{1}{2}}$ $3^{\frac{3}{2}}$

$l)\ \left(-64^{\frac{4}{3}}\right)^{\frac{1}{2}}$ 16

2. Express the following as a single power (Simplify or reduce the base if possible)

a) $\sqrt{15}$ $15^{\frac{1}{2}}$

b) $\sqrt[3]{20}$ $20^{\frac{1}{3}}$

c) $27^{0.\overline{3}}$ $= 27^{\frac{1}{3}}$ 3

d) $32^{1.2}$ $= 32^{\frac{6}{5}}$ 64

e) $\sqrt[3]{-9}$ $(-(3^2))^{1/3}$ $-3^{2/3}$ $\sqrt[3]{-3^2}$ $-3^{\frac{2}{3}}$

f) $\sqrt[3]{111}$ $111^{\frac{1}{3}}$

g) $\sqrt[3]{81}$ $= 3^{\frac{4}{3}}$ $3^{\frac{4}{3}}$

h) $\sqrt[3]{128}$ $= 2^{\frac{7}{3}}$ $2^{\frac{7}{3}}$

$i)(-216)^{\frac{2}{5}}$ $=(-6^3)^{\frac{2}{5}}$ $-6^{3\cdot\frac{2}{5}} = -6^{\frac{6}{5}}$ $=(-6)^{\frac{6}{5}}$ $(-6)^{\frac{6}{5}}$ 0.5

j) $\sqrt[5]{(32)^3}$ $8 = 2^3$

$k)\ (36)^{-\frac{7}{5}}$ $= 6^{2\cdot -\frac{7}{5}} = 6^{-\frac{14}{5}}$ $6^{-\frac{14}{5}}$

$l)\ \dfrac{1}{(-8)^{-\frac{2}{3}}}$ $4 = 2^2$

m) $\left(\dfrac{1}{16}\right)^{\frac{-1}{2}}$ $4 = 2^2$

n) $\left(-\dfrac{1}{27}\right)^{\frac{-2}{3}}$ 9

p) $\left(\dfrac{8}{125}\right)^{\frac{2}{3}}$ $\frac{4}{25}$

q) $\left(\dfrac{9}{16}\right)^{\frac{-3}{2}}$ $\frac{64}{27}$

3. Simplify the following:

$a)\ x^2 \times x^{\frac{1}{2}}$ $x^{\frac{5}{2}}$

$b)\ x^2 \times x^{\frac{1}{2}}$ $x^{\frac{5}{2}}$

$c)\ x^{\frac{3}{2}} \times x^{\frac{4}{2}} \times x^{\frac{5}{2}} \div x^{\frac{6}{2}}$ $= x^{\frac{12}{2}} \div x^{\frac{6}{2}}$ x^3

$d)\ x^{\frac{3}{2}} \div x^{\frac{4}{3}}$ $x^{\frac{1}{6}}$

$e)\ \left(\dfrac{1}{x}\right)^{\frac{3}{2}} \div \sqrt[5]{x^4} \div \left(\dfrac{-1}{x^2}\right)^{\frac{5}{3}}$ $= \frac{1}{x^{\frac{3}{2}}} \div x^{\frac{4}{5}} \div \frac{-1}{x^{\frac{10}{3}}} = x^{\frac{11}{6}} \div x^{\frac{4}{5}}$ $x^{\frac{-79}{30}}$

$= \frac{1}{x^{\frac{3}{2}}} \div x^{\frac{4}{5}} \div \frac{-1}{x^{\frac{10}{3}}} = (-1)\frac{1}{x^{-\frac{11}{6}}} \div x^{\frac{4}{5}}$

$= (-1)x^{\frac{11}{6}} \div x^{\frac{4}{5}} = (-1)x^{\frac{31}{30}} = -x^{\frac{31}{30}}$

$f)\ \sqrt{\left(\sqrt{y}\right)^{\frac{3}{2}}(-y)^{\frac{2}{3}}}$ $= \sqrt{y^{\frac{3}{4}}y^{\frac{2}{3}}} = (y^{\frac{17}{12}})^{\frac{1}{2}}$ $y^{\frac{17}{24}}$

$g)\ \sqrt[3]{\dfrac{-8}{x}}$ $\frac{-2}{\sqrt[3]{x}}$

$h)\ \left(x^2 \times x^{\frac{1}{2}}\right)^{\frac{2}{3}} \div x^{\frac{9}{4}}$ $= x^{\frac{10}{6}} \div x^{\frac{9}{4}} = x^{-\frac{7}{12}}$ $x^{-\frac{7}{12}}$

4. Simplify each expression and place them in increasing order:

$A = 256^{\frac{-5}{8}}$ $\quad B = \left(81^{\frac{1}{4}}\right)^3$ $\quad C = \left(\frac{2}{5}\right)^{-3}$ $\quad D = -49^{\frac{3}{2}}$

$= \left(\sqrt[8]{256}\right)^{-5} = \frac{1}{2^5} = \frac{1}{32}$

$= 3^3 = 27$

$= \frac{1}{\left(\frac{2}{5}\right)^3} = \frac{1}{\frac{8}{125}} = \frac{125}{8}$

$= 15.625$

$= -7^3$

$= -343$

$-343 < \frac{1}{32} < 15.625 < 27 \Rightarrow \boxed{D < A < C < B}$

5. Simplify the following:

$a)\ 125^{\frac{2}{3}} \div \left(\sqrt[5]{32}\right)^6 - \sqrt{\sqrt{\frac{1}{256}}}$

$25 \div 64 - \frac{1}{4} = \frac{25}{64} - \frac{16}{64} = \frac{9}{64}$

$\underline{\frac{9}{64}}$

$b)\ \sqrt{\left(\sqrt[3]{64}\right)^2 + \left(\sqrt{81}\right)^{\frac{1}{2}}}$

$\sqrt{16 + 3} = \sqrt{19}$

$\underline{\sqrt{19}}$

$c)\ \left(\sqrt{\sqrt{256}}\right)^{-3} - \left(\frac{1}{-27}\right)^{\frac{1}{3}} \times \left(\sqrt[3]{216}\right)$

$\frac{1}{64} - \frac{1}{-3} \times 6 = \frac{1}{64} - (-2) = 2\frac{1}{64}$

$\underline{2\frac{1}{64}}$

$d)\ \left(32^{0.2} + 27^{-0.\bar{3}} \times 216^{0.\bar{6}}\right)^{-2}$

$\left(32^{\frac{1}{5}} + 27^{-\frac{1}{3}} \times 216^{\frac{2}{3}}\right)^{-2}$

$= \left(2 + \frac{1}{3} \times 36\right)^{-2} = 14^{-2} = \frac{1}{196}$

$\underline{\frac{1}{196}}$

$e)\ \sqrt{(100)^{-1.5} \times (27)^{-1.\bar{3}} \div (6.25)^{-0.5}}$

$\sqrt{100^{-\frac{3}{2}} \times 27^{-\frac{4}{3}} \div \frac{25}{4}^{-\frac{1}{2}}}$

$= \sqrt{\frac{1}{1000} \times \frac{1}{81} \div \frac{2}{5}} = \sqrt{\frac{1}{32400}} = \frac{1}{180}$

$\underline{\frac{1}{180}}$

6. The population of a cockroach colony can be given by the formula: $F = I \times N^{\frac{A}{L}}$. "F" is the final population, "I" is the initial population, "A" is the time to reach the final population, "N" is the rate of growth, and "L" is the time for one cycle of growth to occur. If a colony of 20 cockroaches increases by 8 times every 3 ~~six~~ days, what is the final population after 7 days? $F = I \times N^{\frac{A}{L}}$

$F = 20 \cdot 8^{\frac{7}{3}} = 20 \cdot \left(\sqrt[3]{8}\right)^7 = 20 \cdot 2^7 = 20 \cdot 128 = \boxed{2560}$

7. If the final population of the cockroaches is 2162688 and the initial population was 33, how many days did it take the colony to grow?

$2162688 = 33 \times 8^{\frac{?}{3}}$

$= 33 \times 2^{?}$

$\rightarrow 2^{?} = 65536$

$? = \boxed{16}$

8. If the accumulate amount of an investment is given by the formula: $A = 3000(1.21)^t$, where "t" is the number of years. How much will be accumulated after 3.5 years?

$A = 3000\left(\frac{121}{100}\right)^{\frac{7}{2}} = 3000\left(\frac{11}{10}\right)^7 = 3000 \cdot \frac{19487171}{10000000} = 3 \cdot \frac{19487171}{10000} = \boxed{5846.1513}$

9. If a, b, and c are distinct positive integers such that $abc = 16$ then what is the largest possible value of: $a^b - b^c + c^a$?

$16 = 1, 2, 4, 4, 8, 16$

$a^b - b^c + c^a = 8^1 - 1^2 + 2^8 = \boxed{263}$

10. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five digit palindrome. There are pairs of four-digit palindromes whose sum is a five digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

ABBA + CDDC = 1___1

(9,2) (8,3) (7,4) (6,5)

$9009 + 2002 = 11011$ } 4 pairs

$2\_\_2 + 9\_\_9$ } 8 $\quad 7\_\_7,\ 4\_\_4$ } 8 $\quad 8\_\_8,\ 3\_\_3$ } 8 $\quad 6\_\_6,\ 5\_\_5$ } 8

$8 \times 4 + 4 = 32 + 4 = \boxed{36 \text{ pairs}}$

11. $\frac{1}{2}+\frac{2}{2^2}+\frac{2^2}{2^3}+\frac{2^3}{2^4}+\ldots\frac{2^{2002}}{2^{2003}}+\frac{2^{2003}}{2^{2004}}$ is equal to:

(A) 1002 (B) 501 (C) $\frac{1}{2^{2004}}$ (D) 2004 (E) $\frac{2004}{2^{2004}}$

$= \underbrace{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\ldots\ldots+\frac{1}{2}}_{2004} = 2004 \cdot \frac{1}{2} = \boxed{1002}$

12. 10^{100} is known as a googol. What is 1000^{100} equal to?

(A) 100 googol (B) 3 googol (C) $\text{googol}^{\text{googol}}$
(D) googol^2 (E) googol^3

$1000^{100} = 10^{3(100)} = (10^{100})^3 = \boxed{\text{googol}^3}$

13. If "x' is an integer less than 100, then how many different values of "x" will make the expression $\sqrt{1+2+3+4+x}$ an integer? (list out all the values for "x")

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

$\sqrt{1+2+3+4+x} = \sqrt{10+x}$

x = 6, 15, 26, 39, 54, 71, 90, -1, -6, -9

$4^2-10 \quad 5^2-10 \quad 6^2-10 \quad 7^2-10 \quad 8^2-10 \quad 9^2-10 \quad 10^2-10 \quad 3^2-10 \quad 2^2-10 \quad 1^2-10$

14. A positive integer is called "multiplicatively perfect" if it is equal to the product of its proper divisors. For example, 10 is a multiplicatively perfect since its proper divisors are 1, 2, and 5 and it is true that $1\times2\times5=10$. How many multiplicatively perfect integers are there between 2 and 100?

2 to 100

- prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

(made up of 2 prime numbers)

2 14 = 14
3 9 = 9
5 5 = 5
7 2 = 2

$14 + 9 + 5 + 2 = \boxed{30}$